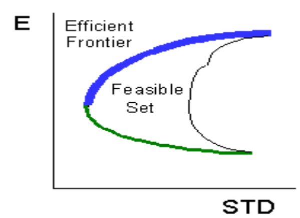
Goodbye to Social Security Benefits



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Term Project

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Table of Contents:

1 Problem and its Importance

The Social Security Trustees estimate that the system's trust fund will be exhausted by 2041, implying that the full Social Security benefits will not be available as promised (CNN Money). This is an alarming estimation which warrants us to think about maximizing the retirement returns so that the benefits offset by the shortage of Social Security funds will be compensated by the extra returns we could ensure through proper balancing of risk/return ratio in the portfolio.

Financial planners estimate that 70-80% of current income will be needed to maintain the current lifestyle in retirement. The notion behind this estimation is that there will not be mortgage payments or work related costs such as commute during retirement though medical expenses go up. Studies indicate that the average life expectancy is going up, and people retiring at age 65 today might live another 20 years, meaning that retirement savings should cover at least 20 years after the retirement.

My wife works for a privately held company and I work for a publicly traded company, both of which offer 401(k) retirement plans as a good employee benefit. Incidentally, both of these employers use Schwab as the fund management service provider and we both have the same set of fund choices to make. There is one difference here, when we do the allocation of the contribution among different funds, the risk profile is different among us. My wife has a very conservative approach toward investing whereas I tend to take more risk for higher returns.

So far, I balanced our portfolios based on higher returns and a little bit of diversification among fund choices based on the fund family. This proved to be futile during 2000-2003 bear market when the mutual funds were yielding negative returns. Our portfolio was reduced to 65% of what it was in year 2000. As an investor with investment experience since 1996, I had seen only the positive side of the market until 2000, but the bear market of 2000-2003 opened my eyes toward the risk associated with these returns and made me think about the risk mitigation through proven techniques. I diversified my investments among different investment choices available through Schwab plan and the returns are above the market returns (S&P 500) since 2003, but I am not sure how this will turn out if the market goes south again. This uncertainty makes me think about ways to measure and quantify the risk associated with my diversified 401(k) portfolio so that I can make informed trade-offs between risk and return.

There are fifteen mutual funds in total among six different fund families available in the Schwab plan. Fund families are already designed by Schwab using Efficient frontier methodology in order to serve the needs of a generic investor just by providing the mean three year annualized return, morning star rating, standard deviation and systematic risk beta (*Appendix A*). I would like to find out if I could do better by diversifying among these fund choices available, while reducing the total risk of the portfolio.

If I try to apply Markowitz's efficient frontier for this portfolio, I need to write a software that could do the number crunching. This could be a major effort for me or major cost associated if I try to buy a service which does this kind of analyses using advanced software programs. The challenge is to find out an optimized way based on mathematical rationale so that the problem can be shrunken to fit into a spreadsheet for easy analyses and could be used repetitively for different investors to fit their investment risk profile. We might want to use this spreadsheet model to rebalance the portfolio periodically as the market conditions change based on the inflation data, employment reports and federal reserve interest rate policies.

2 Solution Procedure and Rationale

I divided the solution procedure into three distinct steps so that one step leads to the next step to accomplish my goal:

1. Data collection and analyses
2. Linear programming formulation to get the fund options reduced into manageable numbers.
3. Markowitz – Efficient frontier plot to optimize the risk return tradeoff.

2.1 Data collection and analyses

I collected basic fund data such as ticker symbol, average annualized return, morning star rating, standard deviation of quarterly returns and systematic risk-beta for all the fifteen funds from the Schwab website and created an Excel spreadsheet table for easy analysis *(Appendix A)*. This table just gave me the risk return data for individual funds but not the data required for the analyses of portfolios that could be created by different level of allocation to different funds. I downloaded the monthly mutual fund price data from the Yahoo! Finance website and created fifteen worksheets so that the covariance matrix could be created *(Appendix B)*. When I have the covariance I can calculate the correlation between all the fund pairs and portfolio standard deviations using the following formulas:

- Covariance *Cov(X,Y)* – Function in MS Excel: COVAR

$$Cov(X,Y) = \frac{\sum (x-\bar{x})(y-\bar{y})}{n}$$

- Correlation coefficient – Function in MS Excel: CORREL

$$r = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i - \bar{Y})^2}} = \frac{\text{cov}(X,Y)}{S_X S_Y}$$

- Portfolio risk – MS Excel doesn't have a general formula to calculate this and this is where the challenge lies. All I need is the Covariance between pair of stocks and standard deviation of individual stocks to come out with portfolio risk.

$$\sigma_P = \sqrt{w^2\sigma_X^2 + (1-w)^2\sigma_Y^2 + 2w(1-w)\,\text{cov}(X,Y)}$$

Unfortunately, this formula becomes more complicated when there are fifteen variables with varying level of allocation 'w' for each. I ended up with 105 co-variances*(Appendix C)*. This lead me to next level of optimization using linear programming so that I can narrow down the funds available to a manageable number in an Excel spreadsheet.

2.2 Linear programming formulation

The objective formula for the linear programming equation is:

- Maximize portfolio return 'R'

$$R = \sum_{i=1}^{n} w_i E_i$$

Where 'w_i' is the percentage allocation of fund *i* and E_i is the average return of the fund *i*.

Similarly the conditions for average morning star ratings, allocation limit for individual fund family and systematic risk, beta are taken as weighted averages and compared against the limits set by the investor.

- Portfolio beta

$$\beta_p = \sum_{i=1}^{n} w_i \beta_i$$ Example of a condition: $0.8 \leq \beta_p \leq 1.2$

- Portfolio morning star rating

$$M^*_p = \sum_{i=1}^{n} w_i M^*_i$$ Example of a condition: $M^*_p \geq 4$

- Also other conditions for limits like total percentage of LargeCap fund family should be more than 20% OR the Bond funds should be at least 5% of the portfolio are included as weighted averages for that fund family and compared against the limits set by the investor.

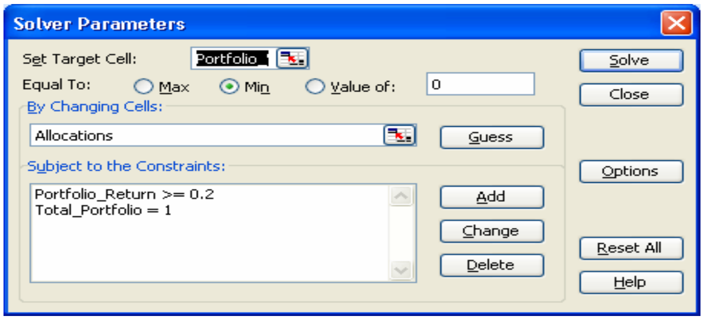
Please note that the systematic risk, beta can not be just the weighted average because the correlation factor might change the whole beta of the portfolio, but this is assumed to be linear at this step just to narrow down the funds available to manageable number of funds. The formulated linear program and conditions are listed in *Appendix D*. Once the linear program is formulated, it can be easily programmed into Excel and solved using the Excel Solver to yield the objective. The programmed Excel sheet is given in the *Appendix E*. After the linear program gives the allocations for different funds, I will pick the funds which got more than 0% allocation and program that into a Markowitz – Efficient frontier model in Excel.

2.3 Markowitz – Efficient frontier graph using Excel

I found that it is easy to start with a readily available Markowitz template from http://www.solver.com website and modify the Visual Basic macros to fit my needs. I downloaded this spread sheet and added the mutual funds I got from step 2 to get a final efficient frontier for the funds and used the risk profile for my wife and me to come out with a final allocation. This spreadsheet requires Variance for each fund, Covariance for all the pairs to calculate the portfolio variance. I already had the Covariance matrix calculated in step1*(Appendix C)*. I could just plug-in the Variance and covariance numbers from step-1 into this spreadsheet and execute the next steps.

We can try different desired portfolio returns and use the Excel Solver to minimize the variance of the portfolio by trying out different allocation levels for each fund. This spreadsheet uses a special macro called ‘QUADPRODUCT’ which allows the computation of quadratic function in one step(*Appendix F*).

- Here is a screenshot of the Solver:



- Fully programmed spreadsheet:

Portfolio Optimization - Markowitz Method

	DODGX	MSSGX	REREX	PTRAX	SWERX	Total
Portfolio %	19.08%	11.12%	21.10%	29.14%	19.56%	100.00%
Expected Return	24.24%	32.57%	31.81%	3.37%	20.75%	
Linear QP Terms	*0*	*0*	*0*	*0*	*0*	

Variance/Covariance Matrix

	DODGX	MSSGX	REREX	PTRAX	SWERX
DODGX	0.90%	0.08%	0.06%	0.00%	0.02%
MSSGX	0.08%	1.99%	0.06%	-0.01%	0.04%
REREX	0.06%	0.06%	1.07%	0.00%	0.03%
PTRAX	0.00%	-0.01%	0.00%	0.18%	0.00%
SWERX	0.02%	0.04%	0.03%	0.00%	0.80%

Variance	0.17%
Std. Dev.	4.09%
Return	20.00%

3 Results and Experience

I wanted to set my conditions for the linear program based on my risk profile as:

Condition Type	Sign	Limit
Return	=	Maximize
Beta Lower Limit	≥	0.5
Beta Higher Limit	≤	1.5
*Rating Lower Limit	≥	3
International Fund Limit %	≤	0.25
Bond Fund Lower Limit %	≥	0
Bond Fund Higher Limit %	≤	0.2
Largecap Lower Limit %	≥	0.2
Smallcap Lower Limit %	≥	0.2
Midcap Lower Limit %	≥	0.2
Stable Higher Limit %	≤	0.1
Total Investment 100%	=	1

Please note that I gave a higher range for beta, between 0.5 and 1.5 and minimum morning star rating average at 3 in a scale of 1-5. This is based on my risk profile. Also I wanted some level of diversification which is reflected in the limits I set for individual fund families.

Excel solver yielded the following result:

R = 28%

Allocations:

DODGX	MSSGX	REREX	SWERX
20%	35%	25%	20%

I did a short interview of my wife after explaining her briefly about the systematic risk and the morning star rating structure. She came out with following conditions:

Condition Type	Sign	Limit
Return	=	Maximize
Beta Lower Limit	≥	0.5
Beta Higher Limit	≤	1.10
*Rating Lower Limit	≥	4
International Fund Limit %	≤	0.15
Bond Fund Lower Limit %	≥	.05
Bond Fund Higher Limit %	≤	0.2
Largecap Lower Limit %	≥	0.2
Smallcap Lower Limit %	≥	0.2
Midcap Lower Limit %	≥	0.2
Stable Higher Limit %	≤	0.1
Total Investment 100%	=	1

Note that she wanted weighted average of morning star rating to be higher than 4 and upper limit of beta to be at 1.10, which shows her higher risk averseness compared to me. She also wanted her international fund limit to be less than 15% as opposed to mine, which was 25%.

Excel solver yielded the following result:

R = 26.57%

Allocations:

DODGX	MSSGX	REREX	PTRAX	SWERX
20%	40%	15%	5%	20%

With these results, I noticed that my wife's portfolio return is expected to be lower by 1.5% than mine due to her risk averse nature. But the most important thing here is that the portfolio funds are narrowed down to five mutual funds diversified among different fund families. The final list of funds that can be programmed into Efficient frontier is:

DODGX	MSSGX	REREX	PTRAX	SWERX

I copied the Covariance and Variance for these funds into the Markowitz model, the spreadsheet is reduced to very concise manageable form. I set the desired level of return at 31.5% so that I can get the full range of returns plotted on the efficient frontier graph starting from minimum return till 31.5%, which is at the higher limit:

Portfolio Optimization - Markowitz Method

	DODGX	MSSGX	REREX	PTRAX	SWERX	Total
Portfolio %	7.45%	33.41%	59.14%	0.00%	0.00%	100.00%
Expected Return	24.24%	32.57%	31.81%	3.37%	20.75%	

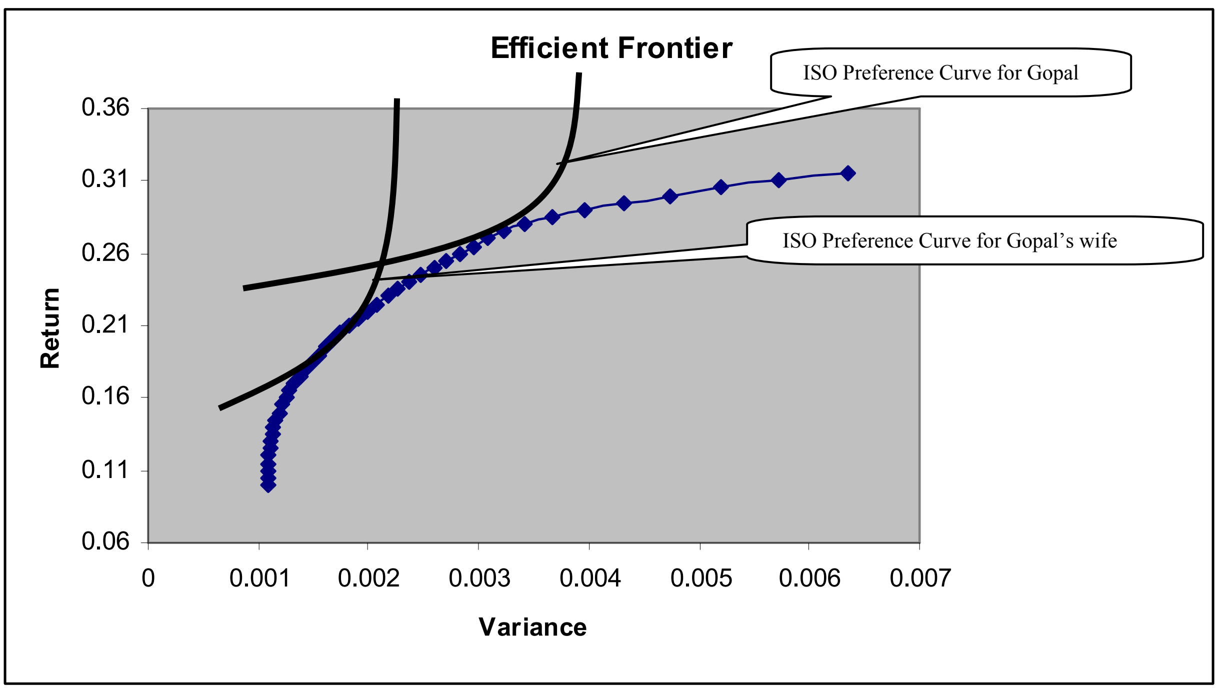
Variance/Covariance Matrix

	DODGX	MSSGX	REREX	PTRAX	SWERX
DODGX	0.90%	0.08%	0.06%	0.00%	0.02%
MSSGX	0.08%	1.99%	0.06%	-0.01%	0.04%
REREX	0.06%	0.06%	1.07%	0.00%	0.03%
PTRAX	0.00%	-0.01%	0.00%	0.18%	0.00%
SWERX	0.02%	0.04%	0.03%	0.00%	0.80%
Variance Terms	0.01%	0.24%	0.39%	0.00%	0.00%
Return Terms	1.81%	10.88%	18.81%	0.00%	0.00%

Create Frontier (Run Only in Desktop Excel; RequiresTools-References Solver checked in VB Editor)

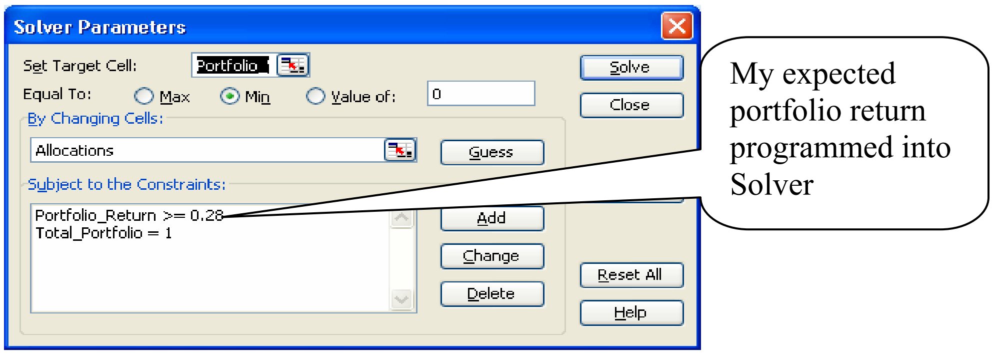
Variance	0.0063504
Std. Dev.	7.97%
Des. Ret	31.50%
Return	31.50%

The efficient frontier formed using portfolio choices available using these funds looked like this:



Based on my subjective analyses, I plotted ISO Preference curve for me and my wife. I noticed that my preference curve touched the efficient frontier curve between 27% to 29% return range whereas my wife's preference curve touched the efficient frontier at around 19% to 21% return range. One important thing I noticed here is that the portfolio risk didn't change even when the portfolio return increased from around 10% to 15%. This will lead me to get the final portfolio allocation for each of us using the Excel solver in the Markowitz model.

I programmed in my portfolio expected return at 28% using Excel Solver:



I got the following portfolio allocation:

Portfolio Optimization - Markowitz Method

	DODGX	MSSGX	REREX	PTRAX	SWERX	Total
Portfolio %	24.03%	19.71%	36.90%	0.00%	19.35%	100.00%
Expected Return	24.24%	32.57%	31.81%	3.37%	20.75%	
Linear QP Terms	*0*	*0*	*0*	*0*	*0*	

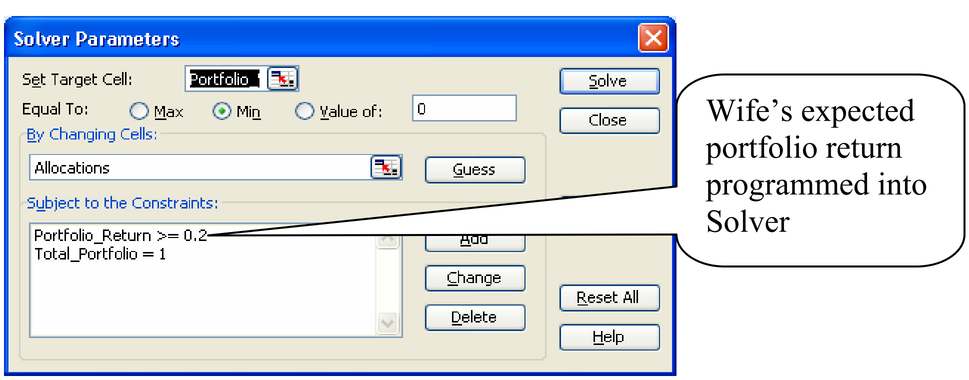
Variance/Covariance Matrix

	DODGX	MSSGX	REREX	PTRAX	SWERX
DODGX	0.90%	0.08%	0.06%	0.00%	0.02%
MSSGX	0.08%	1.99%	0.06%	-0.01%	0.04%
REREX	0.06%	0.06%	1.07%	0.00%	0.03%
PTRAX	0.00%	-0.01%	0.00%	0.18%	0.00%
SWERX	0.02%	0.04%	0.03%	0.00%	0.80%

Variance	0.34%
Std. Dev.	5.84%
Return	28.00%

I noticed that the standard deviation here is 5.84% and the allocated portfolio is changed a little bit compared to the one solved by linear programming. I also noticed that the portfolio risk 5.84% is lower than the S&P 500 market fund *(Appendix A)*, which stands at 8.78% with just 17.05% return whereas my new portfolio is at 28% return.

I programmed the solver for my wife's efficient frontier with expected portfolio return at 20% using Excel Solver:



She got the following portfolio allocation:

Portfolio Optimization - Markowitz Method

	DODGX	MSSGX	REREX	PTRAX	SWERX	Total
Portfolio %	19.08%	11.12%	21.10%	29.14%	19.56%	100.00%
Expected Return	24.24%	32.57%	31.81%	3.37%	20.75%	
Linear QP Terms	*0*	*0*	*0*	*0*	*0*	

Variance/Covariance Matrix

	DODGX	MSSGX	REREX	PTRAX	SWERX
DODGX	0.90%	0.08%	0.06%	0.00%	0.02%
MSSGX	0.08%	1.99%	0.06%	-0.01%	0.04%
REREX	0.06%	0.06%	1.07%	0.00%	0.03%
PTRAX	0.00%	-0.01%	0.00%	0.18%	0.00%
SWERX	0.02%	0.04%	0.03%	0.00%	0.80%

Variance	0.17%
Std. Dev.	4.09%
Return	20.00%

She noticed that the bond fund, PTRAX allocation 29.14% is much different than the linear programming based allocation of 5% and the expected return is down to 20% from the linear programming solve of 26.57%. On the other hand, there is no way to find out the exact risk level of linear program portfolio allocation because PTRAX has negative correlation with some of the funds. Given the standard deviation of 4.09% which is way lower than the S&P 500 fund risk of 8.78%, she was happy with the Markowitz allocation.

4 Critique and Possible Extension

Based on the above observations we rebalanced our 401(k) allocations to fit the Markowitz based solution. This analyses leaves us with peace of mind that we used a scientific approach to do the portfolio allocation. We can also use the same spreadsheets to get the new portfolio balance to suit the change in market conditions or our ISO preference curve.

There is some credence to the argument that what was the purpose of using linear programming, unnecessarily to complicate the approach? But I had no other quantitative rationale to select the right five funds among fifteen available. If I had just used the funds with maximum mean annualized return from each family, I might have missed the morning star rating level and the first level of risk approximation. All these factors make me think that the three step approach I took yielded the optimum portfolio allocation for both of us.

I can extend this model to some of the technology decisions we make at my work place. We use computer hardware supplied by different vendors to run the e-business software. Usually the choice of hardware is made based on the reliability and performance. Performance measure of the computer is usually linear because the performance is directly proportional to the clock speed (MHz) of the CPU. On the other hand the reliability is measure based on the historical failure rates of different components. We usually get the failure statistics such as mean time between failures and its standard deviation from the manufacturer. We deploy each application on multiple computers clubbed together as one logical unit called pool to ensure the capacity to support millions of customers and distribute the load using a specialized hardware called load balancer. If we deploy a mixed portfolio of hardware in each of the pools we might be able to reduce the risk of failures while ensuring the higher performance.

I need to gather all the manufacturer data and the historical failure data from our company records to come out with right statistics to see if this approach can be used as an efficient methodology to allocate hardware for each pool of servers.

5 References

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QUADPRODUCT – Frontline Systems
http://www.frontsys.com/members/dwn_xl97.htm

6 Appendix

A. Data collected from Schwab plan website for individual funds

Code	Fund	Ticker	Morning Star Rating (# of *s)	Mean Annaul Return (3 yr annualized)	SD	Beta
	Large Company					
L1	American Funds Grth Fund of Amer R4	RGAEX	5	22.04%	10.19	1.07
L2	Dodge & Cox Stock	DODGX	5	24.24%	9.51	1.02
L3	Schwab S&P 500 Index Sel	SWPPX	3	17.05%	8.78	1
	Small Company					
S1	Calamos Growth A	CVGRX	4	22.43%	15.32	1.35
S2	Harbor Small Cap Value Ret	HSVRX	4	31.25%	13.15	1.25
S3	Hotchkis and Wiley Mid-Cap Value A	HWMAX	5	31.26%	12.4	1.25
S4	Morgan Stanley Inst Small Co Gr A	MSSGX	4	31.57%	14.12	1.28
	International					
I1	American Funds Capital World G/I R4	RWIEX	4	28.86%	9.3	0.83
I2	American Funds EuroPacific Gr R4	REREX	4	31.81%	10.33	0.92
	Bonds					
B1	PIMCO Total Return Admin	PTRAX	5	3.37%	4.19	1
	Balanced, Managed funds					
M1	Dodge & Cox Balanced	DODBX	5	17%	6.56	0.9
M2	Schwab Managed Retirement 2010	SWBRX	5	11.98%	4.65	0.66
M3	Schwab Managed Retirement 2020	SWCRX	4	16.33%	6.74	0.93
M4	Schwab Managed Retirement 2030	SWDRX	4	19.01%	8.13	0.88
M5	Schwab Managed Retirement 2040	SWERX	5	20.75%	8.97	0.97
	Capital Preservation					
V1	Schwab Stable Value	SWBSV	4	3.87%	0.5	0.03

B. Sample of data collected from Yahoo Finance for monthly return

Date	Open	High	Low	Close	Adj. Close*	Monthly Return
1-May-06	11.46	11.46	11.46	11.46	11.46	-0.00087184
3-Apr-06	11.31	11.49	11.23	11.47	11.47	0.016843972
1-Mar-06	11.16	11.3	10.98	11.28	11.28	0.018050542
1-Feb-06	11.04	11.17	10.88	11.08	11.08	0.004533092
3-Jan-06	10.81	11.04	10.81	11.03	11.03	0.035680751
1-Dec-05	10.81	10.93	10.65	10.65	10.65	-0.003741815
1-Nov-05	10.38	10.75	10.38	10.69	10.69	0.029865125
3-Oct-05	10.56	10.56	10.16	10.38	10.38	-0.017045455
1-Sep-05	10.45	10.57	10.36	10.56	10.56	0.014409222
1-Aug-05	10.39	10.48	10.28	10.41	10.41	0.003857281
20-Jul-05	10.32	10.41	10.26	10.37	10.37	
Fund Return Data for SWREX						

C. Covariance Matrix

Covariance Matrix Using Excel formula COVAR (Data in RED are negative numbers)

	RGAEX	DODGX	SWPPX	CVGRX	HSVRX	HWMAX	MSSGX	RWIEX	REREX	PTRAX	DODBX	SWBRX	SWCRX	SWDRX	SWERX
RGAEX	0	0.000626	0.000536	0.001000	0.000852	0.000759	0.000897	0.000545	0.000576	0.000026	0.000420	0.000217	0.000238	0.000255	0.000275
DODGX	0.000626	0	0.000527	0.000883	0.000782	0.000766	0.000777	0.000555	0.000573	0.000014	0.000437	0.000175	0.000191	0.000207	0.000221
SWPPX	0.000536	0.000527	0	0.000757	0.000674	0.000643	0.000676	0.000455	0.000465	0.000013	0.000355	0.000170	0.000184	0.000205	0.000216
CVGRX	0.001000	0.000883	0.000757	0	0.001316	0.001134	0.001516	0.000753	0.000819	0.000073	0.000580	0.000333	0.000364	0.000397	0.000430
HSVRX	0.000852	0.000782	0.000674	0.001316	0	0.001076	0.001268	0.000676	0.000667	0.000014	0.000531	0.000342	0.000373	0.000407	0.000440
HWMAX	0.000759	0.000766	0.000643	0.001134	0.001076	0	0.001059	0.000652	0.000641	0.000015	0.000520	0.000251	0.000273	0.000300	0.000319
MSSGX	0.000897	0.000777	0.000676	0.001516	0.001268	0.001059	0	0.000633	0.000641	0.000076	0.000513	0.000308	0.000336	0.000373	0.000404
RWIEX	0.000545	0.000555	0.000455	0.000753	0.000676	0.000652	0.000633	0	0.000642	0.000042	0.000384	0.000202	0.000223	0.000240	0.000256
REREX	0.000576	0.000573	0.000465	0.000819	0.000667	0.000641	0.000641	0.000642	0	0.000033	0.000391	0.000250	0.000275	0.000294	0.000313
PTRAX	0.000026	0.000014	0.000013	0.000073	0.000014	0.000015	0.000076	0.000042	0.000033	0	0.000017	0.000009	0.000009	0.000004	0.000001
DODBX	0.000420	0.000437	0.000355	0.000580	0.000531	0.000520	0.000513	0.000384	0.000391	0.000017	0	0.000117	0.000128	0.000138	0.000147
SWBRX	0.000217	0.000175	0.000170	0.000333	0.000342	0.000251	0.000308	0.000202	0.000250	0.000009	0.000117	0	0.000158	0.000173	0.000182
SWCRX	0.000238	0.000191	0.000184	0.000364	0.000373	0.000273	0.000336	0.000223	0.000275	0.000009	0.000128	0.000158	0	0.000186	0.000197
SWDRX	0.000255	0.000207	0.000205	0.000397	0.000407	0.000300	0.000373	0.000240	0.000294	0.000004	0.000138	0.000173	0.000186	0	0.000215
SWERX	0.000275	0.000221	0.000216	0.000430	0.000440	0.000319	0.000404	0.000256	0.000313	0.000001	0.000147	0.000182	0.000197	0.000215	0

D. Linear programming formulation

Problem:	Formula	Condition	Note
Maximize Return:	*22.04 L1 + 24.24 L2 + 17.05 L3 + 22.43 S1 + 31.25 S2 + 31.26 S3 + 31.57 S4 + 28.86 I1 + 31.81 I2 + 3.37 B1 + 17 M1 + 11.98 M2 + 16.33 M3 + 19.01 M4 + 20.75 M5 + 3.87 V1*	Maximum	As much as possible
Star Rating 4 or higher	*5*L1+5*L2+3*L3+4*S1+4*S2+5*S3+4*S4+4*I1+4*I2+5*B 1+5*M1+5*M2+4*M3+4*M4+5*M5+4*V1*	≥ 4	Good quality
Systematic Risk Limits	*1.07L1+ 1.02L2+ 1L3+ 1.35S1+ 1.25S2+ 1.25S3+ 1.28S4+ 0.83I1+ 0.92I2+ 1B1+ 0.9M1+ 0.66M2+ 0.93M3+ 0.88M4+ 0.97M5+ 0.03V*	≥1, ≤1.1	Conservative with little higher risk than market.
International: <= .25	*I1+I2*	≤ 25%	Less than 25%
Bonds: 0<= B <=.20	*B1*	≤ 20%	Less than 20%, Too young for Bonds
Large Company: >=.2	*L1+L2+L3*	≥ 20%	Stability. More than 20%
Small Company: >=.20	*S1+S2+S3+S4*	≥ 20%	More than 20%
Managed: 0<=M<=.20	*M1+M2+M3+M4+M5+M6*	≥ 0%, ≤ 20%	Between 0 and 20%
Value: <=.1	*V1*	≤ 10%	Less than 10% because low yield
Total Investment=100%	*L1+L2+L3+S1+S2+S3+S4+I1+I2+B1+M1+M2+M3+M4+ M5+V1*	1	Fully invested

E. Linear Programming on Excel

E.	Funds Available																		
Condition	RGAEX	DODGX	SWPPX	CVGRX	HSVRX	HWMAX	MSSGX	RWIEX	REREX	PTRAX	DODBX	SWBRX	SWCRX	SWDRX	SWERX	SWBSV	Sign	Condition	Solved LHS
Return	22.04	24.24	17.05	22.43	31.25	31.26	31.57	28.86	31.81	3.37	17	11.98	16.33	19.01	20.75	3.87	=	Maximize	26.57
Beta Higher Limit	1.07	1.02	1	1.35	1.25	1.25	1.28	0.83	0.92	1	0.9	0.66	0.93	0.88	0.97	0.03	≥	0.5	1.10
Beta Lower Limit	1.07	1.02	1	1.35	1.25	1.25	1.28	0.83	0.92	1	0.9	0.66	0.93	0.88	0.97	0.03	≤	1.1	1.10
*Rating Lower Limit	5	5	3	4	4	5	4	4	4	5	5	5	4	4	5	4	≥	4	4.45
International Fund Limit %								1	1								≤	0.15	0.15
Bond Fund Lower Limit %										1							≥	0.05	0.05
Bond Fund Higher Limit %										1							≤	0.2	0.05
Largecap Lower Limit %	1	1	1														≥	0.2	0.20
Smallcap Lower Limit %				1	1	1	1										≥	0.2	0.40
Midcap Lower Limit %											1	1	1	1	1		≥	0.2	0.20
Stable Higher Limit %																1	≤	0.1	0.00
Total Investment 100%	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	=	1	1.00
	RGAEX	DODGX	SWPPX	CVGRX	HSVRX	HWMAX	MSSGX	RWIEX	REREX	PTRAX	DODBX	SWBRX	SWCRX	SWDRX	SWERX	SWBSV			
Solved Values	0%	20%	0%	0%	0%	0%	40%	0%	15%	5%	0%	0%	0%	0%	20%	0%			

F. QUADPRODUCT Function for Excel (*Courtesy: Frontline Systems Inc.*)

QUADPRODUCT is an add-in function, similar to DOTPRODUCT, which allows you to compute a quadratic function in one step. A common use of QUADPRODUCT is in the construction of efficient portfolios, often called portfolio optimization, where the objective is to minimize the total portfolio variance (a measure of riskiness), subject to a threshold on the expected portfolio return. The portfolio variance is a quadratic function of the decision variables (the amounts allocated to each security in the portfolio).

If you sign up (it's free) for our Private Web for Solver Users, you can download QUADPRODUCT and use this function in your own models -- even if you don't have one of our enhanced solvers!

A quadratic function is a sum of terms, where each term is a (positive or negative) constant (called a *coefficient*) multiplied by a *single variable* or the *product of two variables*. This means that in order to represent the most general quadratic function, we might have a coefficient for each instance of a single variable, and a coefficient for each possible *pair* of two variables. The QUADPRODUCT function is designed to supply such coefficients in a manner similar to SUMPRODUCT and DOTPRODUCT.

You supply the arguments of QUADPRODUCT as shown below:

=QUADPRODUCT(*variable cells*, *single coefficients*, *pair coefficients*)

The second argument supplies the coefficients to be multiplied by each single variable in the first argument, using an element-by-element correspondence. The third argument supplies the coefficients to be multiplied by each *pair* of variables drawn from the first argument; hence if there are N cells in the first argument, there must be N*N cells in the third argument. The pairs are enumerated starting with the first cell paired with itself, then the first cell paired with the second cell, and so on. For example, if the first argument consisted of the cells A1:A3, there should be nine cells in the third argument, and the values in those cells will be multiplied by the following pairs in order: A1*A1, A1*A2, A1*A3, A2*A1, A2*A2, A2*A3, A3*A1, A3*A2, and A3*A3. The value returned by QUADPRODUCT is the sum of all of the coefficients multiplied by their corresponding single variables or pairs of variables.

When used with the Solver, the first argument should consist entirely of decision variable (changing) cells. The second and third arguments should consist entirely of cells whose values are *constant* in the optimization problem; if these cells contain formulas, those formulas must not refer to any of the decision variables. For example, in a portfolio optimization problem, the decision variables are the amounts to be allocated to each security; cells in the second argument would all be zero, since no linear terms are needed; and cells in the third argument would contain the covariance of price changes of each pair of securities.

Each of the arguments may be a multiple selection: If an argument consists of more than one area or block of cells, the areas must be separated by commas and the whole argument must be enclosed in parentheses.